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Nonmonotonic causal theories

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Outline

- Introduction
- Syntax and Semantics
- Knowledge Representation
- *C*+ Language
- Theorems

Introduction

This paper

- Defines nonmonotonic causal theories (NCT)
- Investigate “definite” NCT
- Shows how they can be used for knowledge representation
- Introduces action language *C*+ based on NCT
- Compares NCT with related works

Syntax of Causal theory

- **Multi-valued propositional signature:** set of constants, c, and their finite domains Dom(c)

  Note: *i*:c, *i* is nonnegative integer and c is constant.
- **Atom:** c=v where v is in Dom(c)
- **Formula:** a propositional combination of atoms. e.g. Loc_a= table $\wedge$ Loc_b=table

Causal Rules

- **Causal rule:**

  $F \Leftarrow G$, where F and G are formulas.

  Causal theory: a set of causal rules.

Semantics (Informal)

- $F \Leftarrow G$: if G is true then there is a cause for F to be true.

  Note that $F \Leftarrow \top$ means that F is caused to be true while $\bot \Leftarrow \neg F$ says that F is true.

- ***F is true in causal theory T if and only if F is caused by T***

Examples

- Let c, d be boolean constants
- $T_1 = \{c{=}t \Leftarrow \top.\ \ d{=}t \Leftarrow c{=}\ t.\ \}$

  Does T_1 entail d=t? Yes.
- $T_2 = \{\bot \Leftarrow \neg c{=}\ t.\ \ d{=}t \Leftarrow c{=}\ t.\ \}$

  Does T_2 entail d=t? No.

Semantics

- An ***interpretation*** is a function that maps constants to elements of their domains.
- An interpretation I ***satisfies*** an atom $c = v$ (symbolically, $I \vDash c=v$) if $I(c) = v$.
- A ***model*** of a set of formulas is an interpretation that satisfies all formulas.

Semantics

- $\boldsymbol{T^I}$: The reduct T^I of T relative to I is the set of the heads of all rules in T whose bodies are satisfied by I.
- I is a model of T if I is the unique model of T^I

Examples

1. $\sigma = \{ c \}$, Dom (c) = { 1, 2, 3 }
T_1: $c = 1 \Leftarrow c = 1$
I(c) = 1; $T^I = \{ c = 1 \}$.

2. $\sigma = \{ c, d \}$, Dom (c) = { 1, 2, 3 }; Dom(d)={t, f}
T_2: $c = 1 \Leftarrow c = 1$
I(c) = 1; I(d) =t; $T^I = \{ c = 1 \}$ *note: I is not the **unique** model of T^I*

3. $\sigma = \{ c \}$, Dom (c) = { 1, 2, 3 }
T_3: $c = 1 \Leftarrow c = 1$. $c = 2 \Leftarrow \top$.
I (c) = 2; $T^I = \{ c = 2 \}$.

4. $\sigma = \{ c \}$, Dom (c) = { 1, 2, 3 }
T_4: $c = 1 \Leftarrow c = 1$. $c = 2 \Leftarrow c = 2$.
a. I (c) = 1; $T^I = \{ c = 1 \}$.
b. I (c) = 2; $T^I = \{ c = 2 \}$.

Definite theories

- Definition:

  A causal theory T is definite if
  - The head of every rule of T is an atom or $\bot$
  - No atom is the head of infinitely many rules of T

Completion Process

- There is a completion process that reduces the problem of finding a model of a definite causal theory to the problem of finding a model of a set of formulas.

Completion Process

- For each atom A whose domain is not a singleton, add formula

  $A \equiv G_1 \vee \ldots \vee G_n$ where $G_1, \ldots, G_n$ are the bodies of the rules of T with head A.

  (Do nothing for singleton formula)

  if n=0, add formula $A \equiv \bot$.

- For each constraint $\bot \Leftarrow F$, add formula $\neg F$.

Example of Completion

- Causal Theory T:

$\sigma = \{ c, d \}$, $Dom(c) = \{ 1,\ldots,n \}$

$Dom(d)=\{t, f\}$

$c=1 \Leftarrow c=1$
$c=2 \Leftarrow c=2$
$c=1 \Leftarrow d=t$
$c=3 \Leftarrow \top$
$d=t \Leftarrow d=t$

$c=1 \equiv c=1 \lor d=t$
$c=2 \equiv c=2$
$c=3 \equiv \top$
$c=v \equiv \bot \quad (v \in Dom(c)\setminus\{1,2,3\})$
$d=t \equiv d=t$
$d=f \equiv \bot$

Theorem

- The models of a definite causal theory are precisely the models of its completion.

Knowledge Representation

- Default reasoning
- Actions and changes
- Example: monkey and bananas

Default

- $c{=}v \Leftarrow c{=}v$ can be used to express the default knowledge. Example:

  $\sigma = \{ c \}$, Dom (c) = $\{ 1, \ldots, n \}$

1. T_1: $c = v_1 \Leftarrow c = v_1$

   $T_1 \models c = v_1$

2. T_2: $c = v_1 \Leftarrow c = v_1$. $c = v_2 \Leftarrow \top$.

   $T_2 \models c = v_2$; $T_2 \models c \neq v_1$

Action and changes

$1 : p \Leftarrow 0 : a$

$0 : p \Leftarrow 0 : p.$

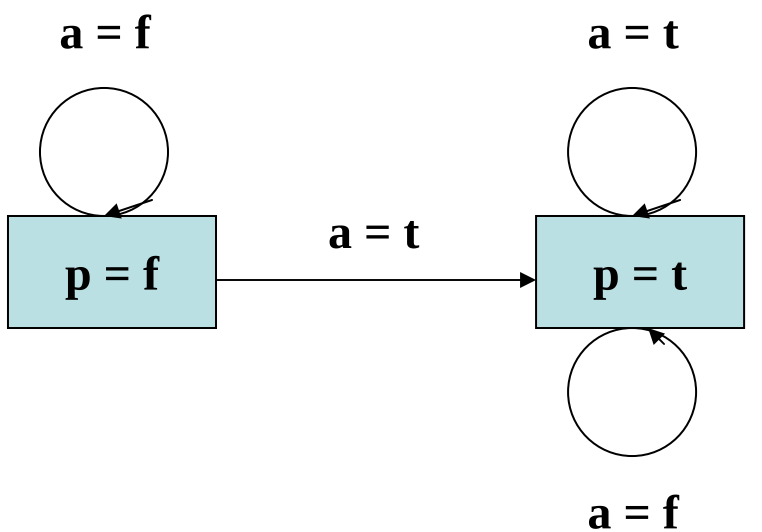
$0 : \neg p \Leftarrow 0 : \neg p$

$0 : a \Leftarrow 0 : a.$

$0 : \neg a \Leftarrow 0 : \neg a$

$1 : p \Leftarrow (0 : p) \wedge (1 : p)$

$1 : \neg p \Leftarrow (0 : \neg p) \wedge (1 : \neg p)$



The Models of the example

- M1={ 0:p=t , 0:a=t, 1:p=t}
- M2={ 0:p=t, 0:a=f, 1:p=t}
- M3={ 0:p=f, 0:a=t, 1:p=t}
- M4={ 0:p=f, 0: a=f, 1:p=f}

If we use i and i+1 instead of 0 and 1, for the length m (i<m), we will have 2^{m+1} models

Monkey and Banana (MB)

- *Signature:*

i : Loc(x), where x $\in$ *{ Monkey, Bananas, Box}*

i : HasBananas, i : OnBox

i : walk, i : climbOn

i : PushBox(l), where l $\in$ *{ L1, L2, L3}*

i : climbOff, i : GraspBananas

MB con'd

- $i{+}1 : Loc(Monkey){=}l \Leftarrow i : Walk(l)$.
- $i{+}1 : HasBananas \Leftarrow i : GraspBananas$.
- $\perp \Leftarrow i : (GraspBananas \wedge \neg OnBox)$.
- $\perp \Leftarrow i : (GraspBananas \wedge Loc(Monkey) \neq Loc(Banans))$.
- $i{+}1 : Loc(Box){=}l \Leftarrow i : PushBox(l)$.
- $i{+}1 : Loc(Monkey){=}l \Leftarrow i : PushBox(l)$.
- ……

Reasoning and planning

- Prediction:

  Given the complete MB theory, we know, initially the monkey is at L_1, the bananas are at L_2, and the box is at L_3. The monkey walks to L_3 then pushes the box to L_2.

  Are the bananas and the box at the same location?

  $[(0{:}Loc(monkey)=L_1 \wedge (0{:}\ Loc(Bananas)=L_2) \wedge (0{:}Loc(Box)=L_3) \wedge (0{:}Walk(L_3)) \wedge (1{:}PushBox(L_2))] \rightarrow 2{:}(Loc(Monkey)=Loc(Bananas) \wedge Loc(Bananas)=Loc(Box))$.

Reasoning and planning

- Postdiction

  The monkey walked to location L_3 and pushed the box. Does it follow that the box was initially at L_3

  $[(0{:}Walk(L_3) \wedge (1{:} \vee PushBox(l)] \rightarrow 0{:}Loc(Box)=L_3$

Reasoning and planning

- Planning

Initial states:

0:Loc(Monkey)=L_1*, 0:Loc(Bananas)=*L_2*, 0:Loc(Box)=*L_3

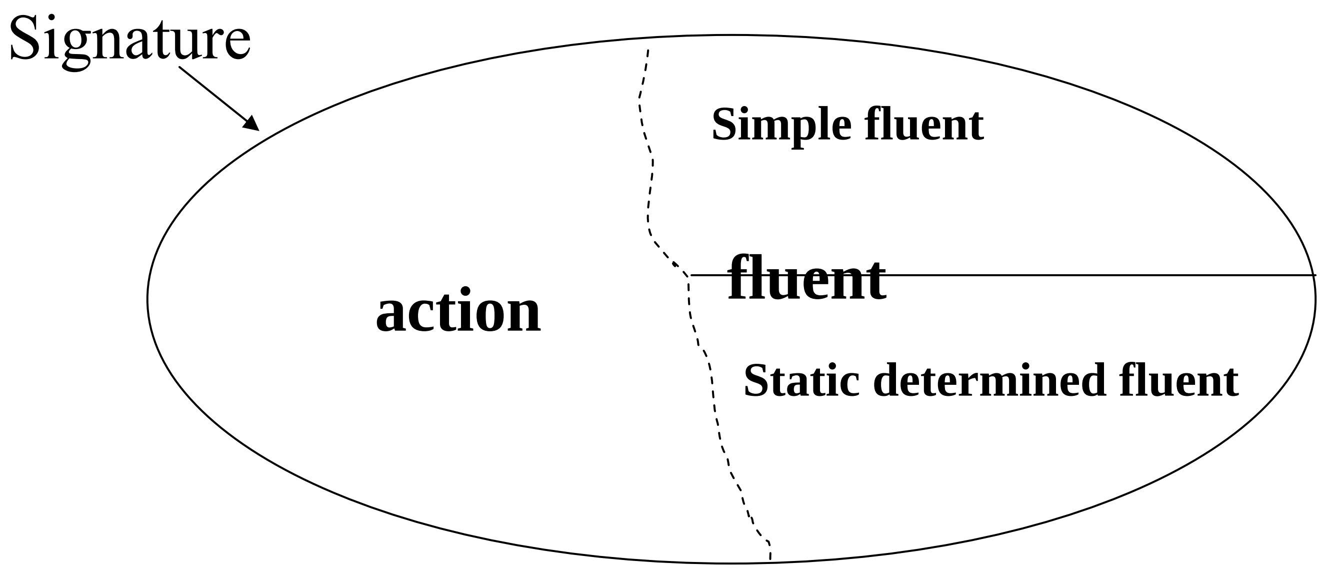
- Goal:

m:HasBanans

- The shortest plan:

0:Walk(L_3*), 1:PushBox(*L_2*), 2:ClimbOn, 3:GraspBananas.*

Definitions of *C*+



Static determined fluent: only appears at the head of static causal laws
Fluent formula: all constants occurring in it are fluent constants.
Action formula: at least one action constant and no fluent constants.

Syntax in C+

- Static laws : ***caused F if G***
- Action dynamic law : ***caused A if H***
- Fluent dynamic law: ***caused F if G after H***
- ***exogenous c***
- ***inertial p***

where F and G are fluent formulas, A is an action formula, H is a formula, c is an action constant, p is a simple fluent constant

Translation

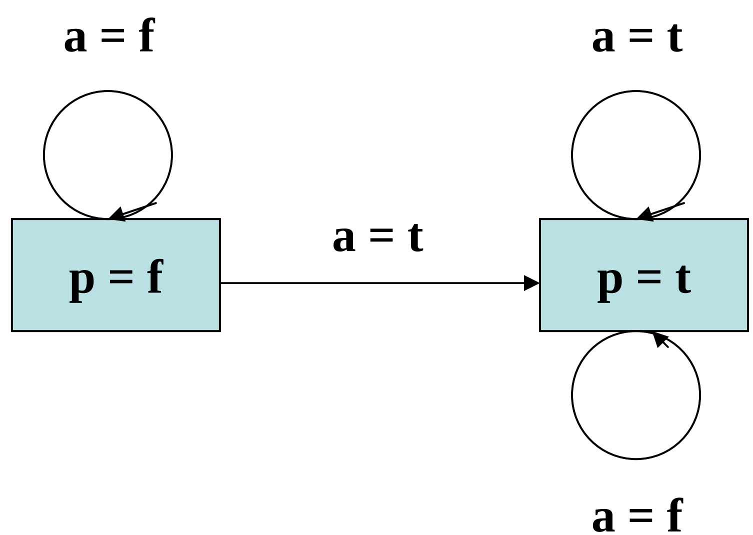
- ***caused F if G*** → $\mathbf{i{:}F} \Leftarrow i{:}G$
- ***caused F if G after H*** → $\mathbf{i{+}1{:}F} \Leftarrow i{+}1{:}H \wedge i{:}G$
- ***exogenous c*** → $\mathbf{i{:}c{=}v} \Leftarrow i{:}c = v$
- ***inertial p*** → $\mathbf{i{+}1{:}p{=}v} \Leftarrow i{+}1{:}p{=}v \wedge i{:}p{=}v$
- *For every simple fluent constant c and every v in Dom(c). Add* $0{:}c{=}v \Leftarrow 0{:}c{=}v$

Example of C+

a causes p

exogenous a

inertial p



Theorems

- *An interpretation I is a model of a causal Theory T if and only if, for every formula F,*

$$I \models F \text{ iff } T^I \models F$$

- *If a causal theory T contains a causal rule* $F \Leftarrow G$ *then T entails* $G \supset F$.

Theorems

- *Let* T_1 *and* T_2 *be causal theories of a signature* σ *such that every rule in* T_2 *is a constraint. An interpretation of* σ *is a model of* $T_1 \cup T_2$ *iff it is a model of* T_1 *and does not violate any of the constraints in* T_2
- *The models of a definite causal theory are precisely the models of its completion.*